

PHOTOGRAPHY OF THE MOON FROM A LUNAR SATELLITE

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"PHOTOGRAPHY OF THE MOON FROM A LUNAR SATELLITE"

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I. Summary

High resolution lunar photography capable of one foot ground resolution has been preliminary-studied to determine the optical requirements.

It has been determined that one major limitation would be the orbiting altitude of the surveying vehicle.

It has been estimated that a lunar satellite orbiting at a 50 nautical mile altitude from the moon's surface would require, as a minimum, a diffraction-limited lens of 9.0 inches diameter and film capable of resolving 225 lines per millimeter.

II. Introduction

In the event that for future missions, a one foot ground resolution is required, this report outlines the optical needs. One foot resolution should not be construed as a limiting value, but as a realistic obtainable goal. The analysis lends itself to any reasonable resolution requirements. It is assumed that film will be used and will be processed by highly skilled methods.

III. Analysis

A. Ground Resolution

The major factors that enter into the determination of the ground resolution of an airborne photographic mission are the altitude and velocity of the vehicle, and the combined lens and film resolution of the photographic system.

The Modulation Transfer Function (MTF) method does not readily permit an analytic solution; therefore, an approximate method is considered. One approximate method is to take the square root of the sum of the squares of the component resolution^{1,2} or:

$$R = \sum_{i=1}^n \sqrt{R_i^2} \quad i = 1, 2, 3, \dots, n \quad (1)$$

The angular resolution of a lens due to diffraction limitation can be shown by a Gaussian approximation of a line-spread function curve, to be roughly²

$$\theta = \frac{0.45 \lambda}{D} \quad (2)$$

and the linear resolution in the image plane is:

$$r_L = F\theta \quad (3)$$

where:

θ = resolution in radians

λ = principal wavelength of light

D = lens diameter

r_L = linear resolution on focal plane of lens

F = focal length

Combining eqs (2) and (3) one obtains:

$$r_L = \frac{0.45F\lambda}{D} \quad (4)$$

To determine the effective ground resolution, R_L , it is necessary to determine the scale factor S , which is a function of the height of the vehicle above the terrain and the focal length of the lens or:

$$R_L = r_L S \quad \text{where } S = \frac{H}{F} \quad (5)$$

Incorporating eq (5) into eq (4) we have:

$$R_L = \frac{0.45\lambda H}{D} \quad (6)$$

It is shown by the above equation that the ground resolution is independent of the focal length of the system.

B. Film

The ground resolution due to the film is the scale factor divided by the stated film resolution or:

$$R_f = \frac{S}{N} \quad (7)$$

where:

R_f = resolution of film

S = scale

N = number of lines/unit distance

Combining eq (5) into eq (7) we have:

$$R_f = \frac{H}{NF} \quad (8)$$

Now the focal length is determined by the lens diameter and its f/number, or

$$f = \frac{F}{D} \quad (9)$$

Hence, we can now consider the ground resolution of the film also as a function of the focal ratio

$$R_f = \frac{H}{NfD} \quad (10)$$

C. Velocity

The ground resolution is also affected by the relative ground motion during the time that the camera shutter remains open. If the photographic film were normal to the ground motion and the camera shutter 100 per cent efficient, the ground resolution due to velocity would be

$$R_v = vt \quad (11)$$

Since a camera shutter takes a definite time to open and close, the actual time can be approximated by a Gaussian curve and evaluated to be²

$$t = t_o / (2\pi)^{1/2} \quad (12)$$

where t_o is the apparent time of shutter speed.

The velocity component is considered to be parallel to the ground. Since there is a field angle ϕ between a line on the ground and the normal line of flight, the velocity component is considered to be

$$V = V_o \sin \phi \quad (13)$$

where V_o is the vehicle velocity.

To evaluate the average value of $\sin \phi$ between 0 and $\pi/2$, we have

$$\int_0^{\pi/2} \frac{\sin \phi \, d\phi}{\pi/2} = \frac{2}{\pi} \quad (14)$$

The time of exposure can be shown to be a function of the exposure required of the film (or ASA rating), the surface brightness and the square of the effective focal ratio or³

$$t_o = \frac{4}{\pi} \frac{f^2}{BZ} \quad (15)$$

where

t_o = required exposure time (sec)

f = effective focal ratio

B = surface brightness - candles/ft.²

Z = ASA film rating

Combining eq's 12-15 into eq 11, we have

$$R_v = \frac{4}{\pi} \cdot \frac{2}{\pi} \cdot \frac{1}{2\pi} \cdot \frac{1}{2} \cdot \frac{f^2 V_o}{BZ} = \frac{.323 f^2 V_o}{BZ} \quad (16)$$

D. System Resolution

It will now be shown how ground resolution capabilities of an aerial photographic reconnaissance system are not only dependent upon the lens and film but also on velocity and altitude. It is fully expected that image motion compensation (IMC) techniques will be used to reduce the smear due to velocity as much as possible but due to limitations of mechanical design and full knowledge of

altitude and velocity, there will be a small residual of smear. A residual smear of 0.5 per cent has been obtained in modern high altitude photography. Assuming a lunar orbital vehicle, traveling at a velocity of the order of 5200-5500 feet/sec (see Fig. 1), less than 1/5000 sec exposure is required for 1 foot resolution. This is beyond the capability of low speed film used with high aperture optics. On the other hand, it is possible to use IMC techniques so that the film moves relative to the ground at only 0.5 per cent of the vehicle velocity which would be 25 ft/sec. This reduction of apparent velocity allows a longer exposure time so that if the exposure time is now slightly smaller than 1/25 sec., the ground resolution due to velocity smear is a little less than one foot but a higher resolution film can be used.

Combining the resolution equations (6), (10), and (16) into eq. (1), we have

$$R_t = \sqrt{\left(\frac{0.45\lambda H}{D}\right)^2 + \left(\frac{H}{ND}\right)^2 + \left(\frac{.323f^2 v_o}{BZ}\right)^2} \quad (17)$$

The quadratic form of R_t with respect to the focal ratio indicates that the equation has a minimum R_t if all of the other parameters are held constant. If we differentiate R_t with respect to f and set it to zero, we find that R_t minimum exists when:

$$f = \left(\frac{1}{2}\right)^{1/6} \left(\frac{H}{ND} \bigg/ \frac{.323V_o}{BZ}\right)^{1/3} \quad (18)$$

Incorporating the value of f into the ground resolution equation (17), we find:

$$R_t = \sqrt{\left(\frac{0.45\lambda H}{D}\right)^2 + 2.18 \left(\frac{H}{ND}\right)^{4/3} \left(\frac{0.323V_o}{BZ}\right)^{2/3}} \quad (19)$$

Kodak Special High Definition Aerial film type So-243 now designated as code 4404 will be the film selected. This film was chosen because of its high resolution.

The characteristics of So-243 film pertinent to this report are:

Resolution = 475 lines/mm - T.O.C. 1000:1

(subject contrast ratio)

Resolution = 200 lines/mm - T.O.C. 1.6:1

(subject contrast ratio)

Exposure Index (ASA Rating) = 3

Laboratory studies show that it is possible to exceed the exposure index by a factor of 10 and to compensate for the light loss by a slow development, therefore an exposure index of 30 will be used for this report. The contrast ratio of the moon will be considered to have an average ratio of 2:1; thus, we will assume a film resolution capability of 225 lines/mm.

The solar illumination in trans-lunar space has been measured to be approximately 13,600 foot-candles, and the average albedo of the moon as measured photographically by Fieldings in 1961 is 10 per cent⁵.

Converting illumination into surface brightness by the relation

$$B = \frac{KE}{\pi} \quad (20)$$

where

B = Surface Brightness Candles/ft²

E = Illumination foot-candles

K = Reflectivity of surface 0.10

One finds the average surface brightness of the moon for photographic purposes to be 432 candles/ft².

Some additional assumptions are required to find the ground resolution of equation (19). The principle wavelength of light (λ) will be assumed to be 5.55×10^{-4} mm. Other losses due to imperfect focussing, vibration, temperature and pressure variation, and improper film processing, etc., have not been considered in this report.

If we treat the resolution factors separately as a function of focal ratio, we see in Figure 2 that the lens resolution remains constant and that the film resolution becomes smaller (better) with increased f/no. Also, due to the increase in length of the required exposure, an increase of image motion compensation reduces (improves) the resolution. The purpose of Figure 2 is to show that a minimum (best) resolution exists. Without image motion compensation, the optimum f/no would be about 4, but with 99.5 per cent IMC, the optimum f/no is about 28. Figure 2 also points out that even if better IMC was available, we could not exceed the lens resolution.

Using the resolution equation (17) to determine ground resolution as a function of focal ratio (see Figure 3), one can find the effect of image motion compensation on performance. Although there is an IMC factor of zero per cent involved as compared to 99.5 per cent IMC, the performance is improved only by a factor of 2.4. The small size of the improvement is due to the fact that the reduction of ground resolution due to velocity has only a partial effect on the entire equation. This partial effect is also to be considered when the altitude is reduced by 50 per cent (see Figure 4). One expects to find the optimum ground resolution to be improved by a factor of 2, instead there is only a 29 per cent performance improvement due to the lower altitude for the conditions given in the Figure.

If the optimum focal ratio (eq. 19) is used, one may plot ground resolution R_t as a function of lens diameter for various altitudes (see Figure 5). The results indicate that if a one foot ground resolution is required, the flight altitude should be about 50 nautical miles, and a lens diameter about 9". The curves in Figure 5 also indicate that increasing the lens diameter does not proportionally increase the resolution performance. Again, it is pointed out that as the limitation due to one of the resolution factors (lens, film, or velocity) is approached, the total theoretical resolution can be approached but not exceeded.

The causation for the optimization that exists can be explained by the factor that increasing focal length with a fixed lens diameter also increases the focal ratio. If we keep increasing our focal length or focal ratio, the image will be dark, and the ground resolution will be poor. If on the other hand, we open our shutter for a long exposure time, we are increasing the blur caused by the velocity component. Either way, a minimum resolution value exists for a given operating condition with respect to film, lens diameter, and velocity.

IV. Additional Parameters

A. Film Weight

Another consideration for determining the weight of the system is the weight of the film. For a first approximation, it is assumed that from a gross photographic survey that there will be approximately 10 possible lunar landing sites, each of a hundred square miles, or 1000 square miles to be closely scanned.

To determine the film equivalent of 1000 square miles, it is necessary to consider the scale factor. Eq. (5) points out that the scale factor is the ratio of altitude to the focal length.

Assuming that the focal length is 113 inches and the altitude is 50 miles, then one linear mile on the ground will mean 2.27 inches of film or a total of approximately 5.15×10^6 square inches of film to photograph 1000 square miles of terrain. An additional 20 per cent should be added to allow for film edge and space between frames

bringing the total to approximately 6.2×10^6 square inches of film. Assuming the thickness of the film is .0032 inches and the specific weight of the cellulose film is .050 lbs/in.³, then the weight of the film would be about 1000 lbs or a pound of film per square mile of terrain.

B. Exposure Time

The time of exposure as expressed in eq (15) was a function of the film rating, the surface brightness and the effective focal ratio or

$$t_o = \frac{4f^2}{\pi BZ} \quad (21)$$

The surface brightness of the moon is considered to be 432 candles/ft.² and the film rating of So-243 is 30. The effective focal ratio can be considered approximately 12.5, thus the time of exposure would be approximately 1/60 second.

C. Image Motion Compensation

The amount of motion of the film to compensate the apparent ground motion would be the ground velocity divided by the scale factor or

$$IMC = V \frac{F}{H} \quad (22)$$

If the same assumptions are used as previously, 50 nautical miles altitude and 113 inch focal length, then the IMC required would be approximately 2 inches per second.

The angular correction would simply be the true ground speed divided by the altitude above the terrain or $5360/30400 = 0.177$ rad/sec.


The amount of image motion compensation required is feasible with present state of the art.⁴

D. Format Size

It has been considered previously that a one foot ground resolution was desirable for the Lunar Photographic Mission. Since weight is always a prime consideration, it is advisable to select the smallest lens possible for a given mission as the system weight increases approximately proportionally to the cube of the lens diameter. An examination of Figure 5 shows that the condition of less than one foot ground resolution can be met with lens diameter of 9" and that the altitude of the vehicle is 50 nautical miles. The optimum focal ratio for the given set of conditions is $f/12.5$ and its optimum focal length is 113 inches. Since we are traveling 1 mile per sec per second, and 1 mile covers 2.25" on the film, one could use 70 mm film with a square format.

IV. Conclusions and Recommendations

It is concluded that a high resolution photographic mission of the lunar surface is feasible within the present state-of-the-art. It is also concluded that a photographic system using film will allow a one-foot ground resolution to be obtained from a 50 nautical mile orbiting satellite.



Sidney Wanger
Member of the Technical Staff

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VELOCITY FOR CIRCULAR LUNAR ORBITS

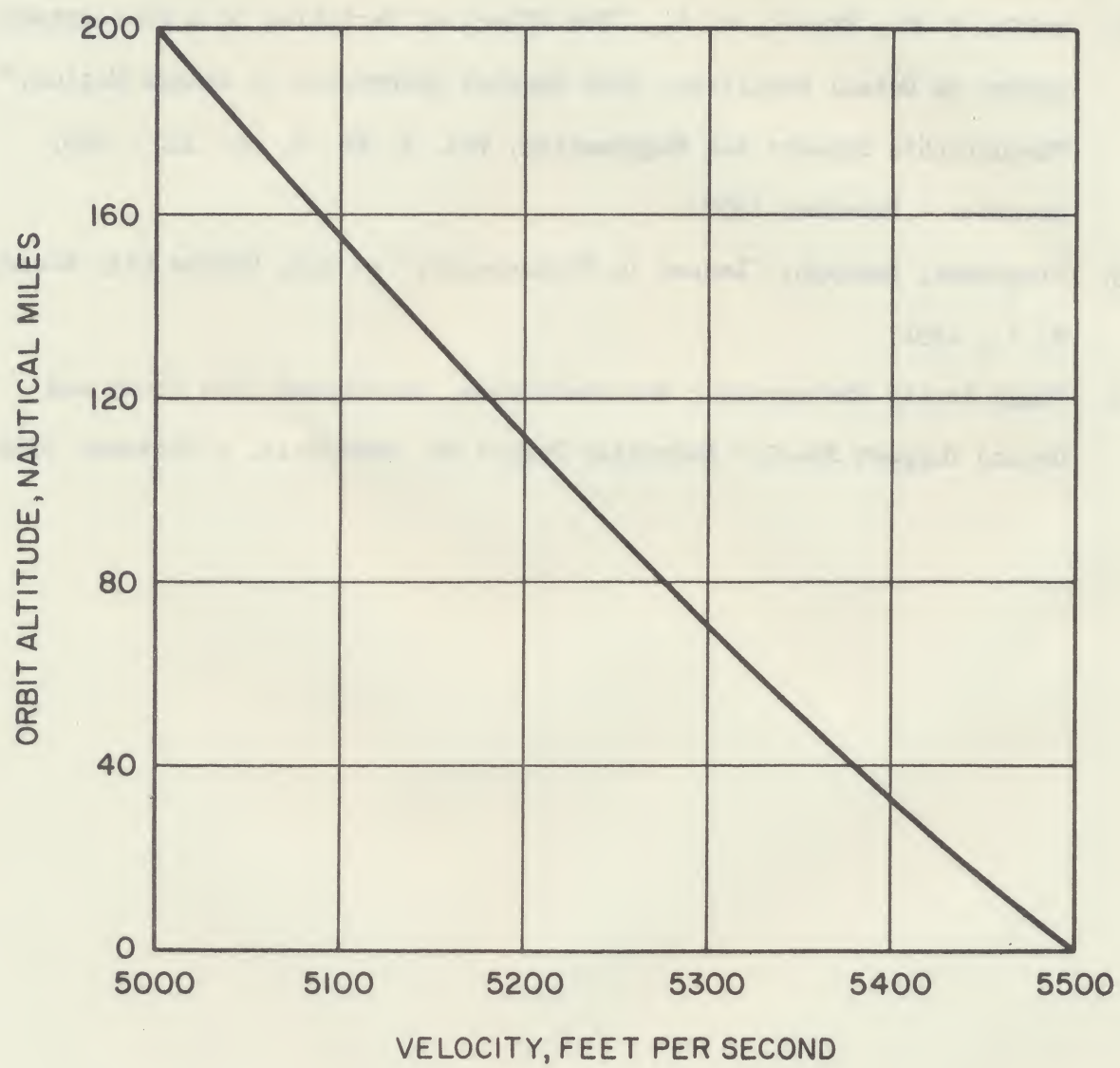


FIGURE 1

PARAMETERS OF AERIAL PHOTOGRAPHY

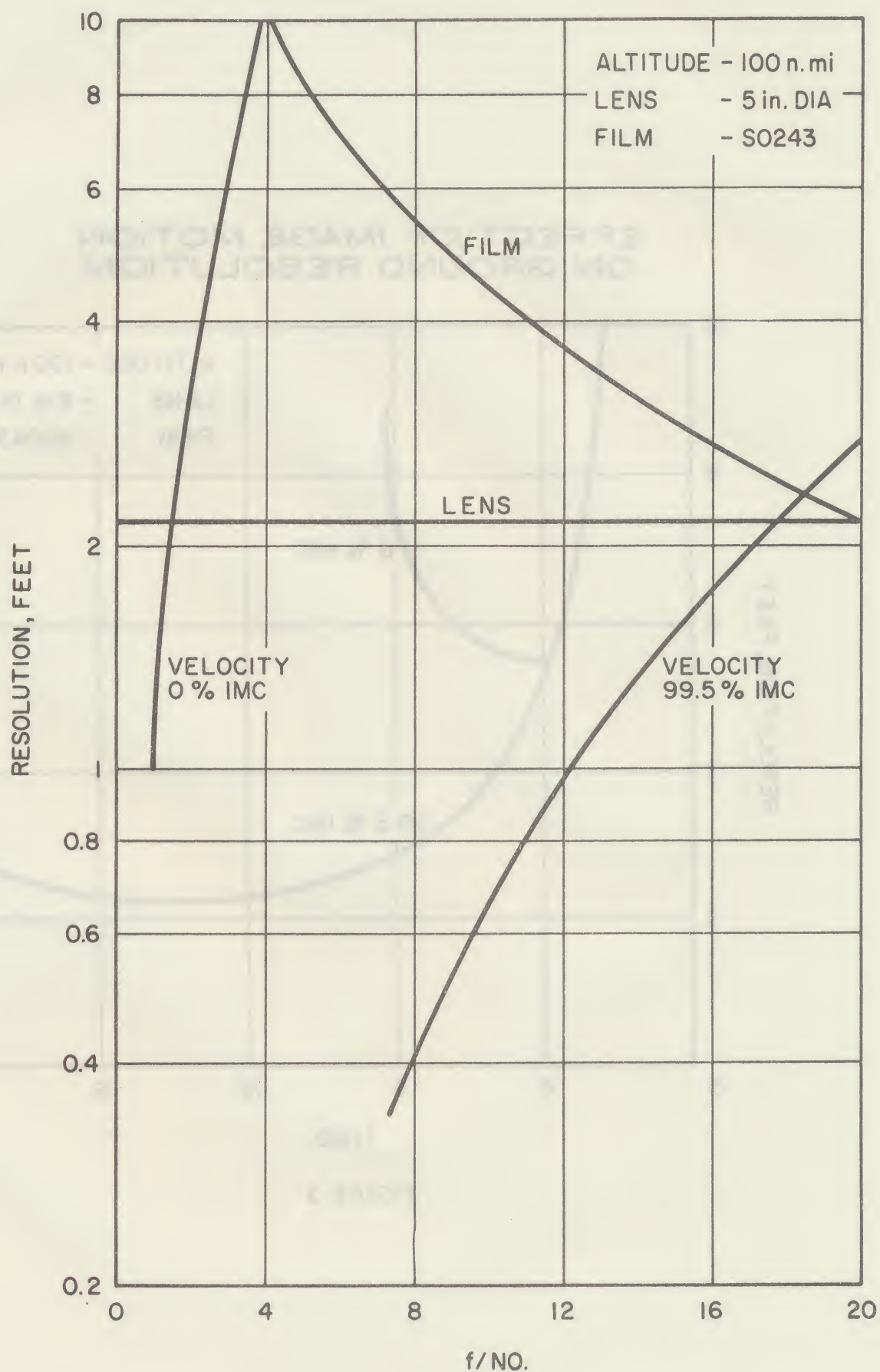


FIGURE 2

EFFECT OF IMAGE MOTION ON GROUND RESOLUTION

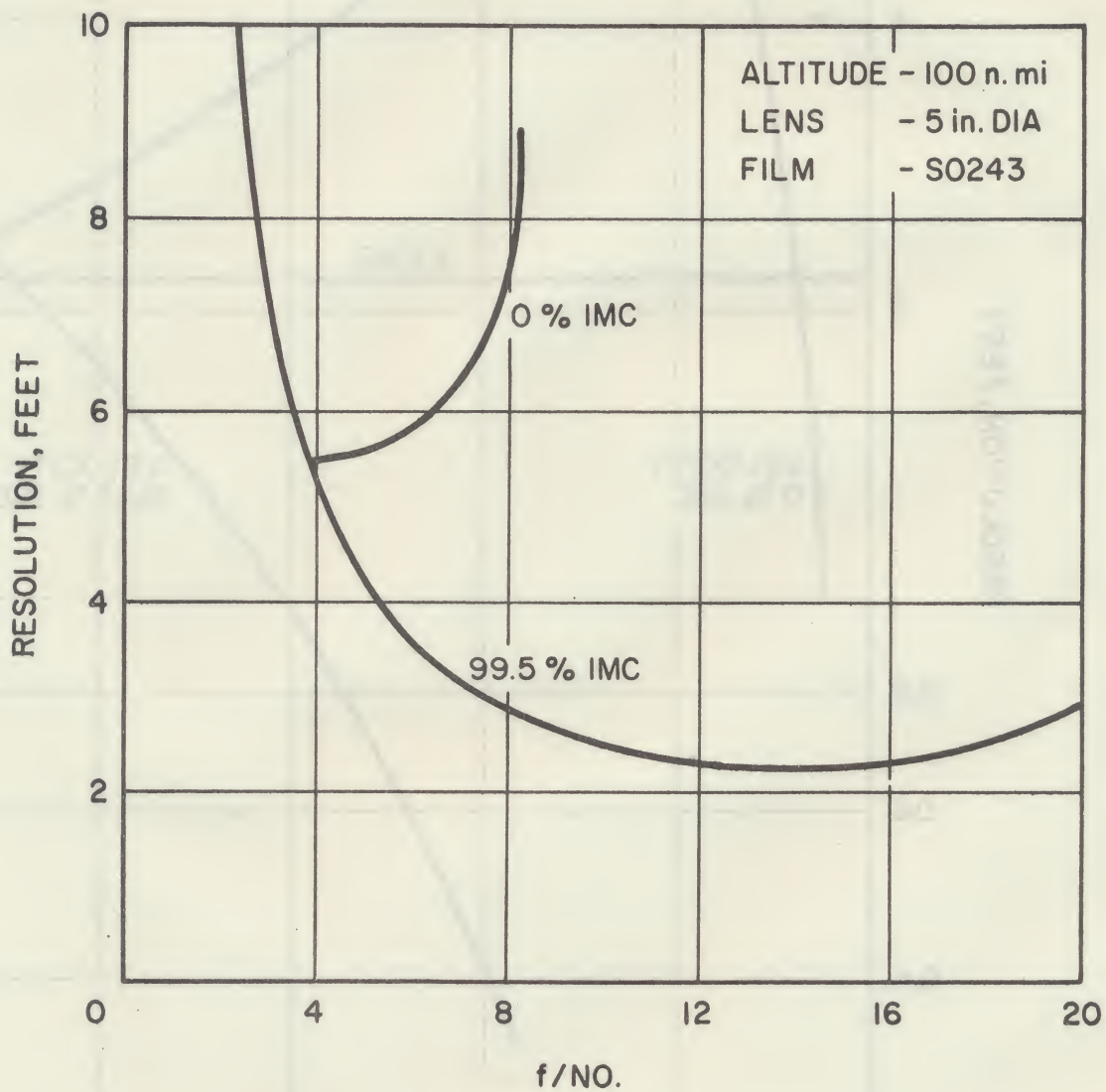


FIGURE 3

EFFECT OF ALTITUDE ON GROUND RESOLUTION

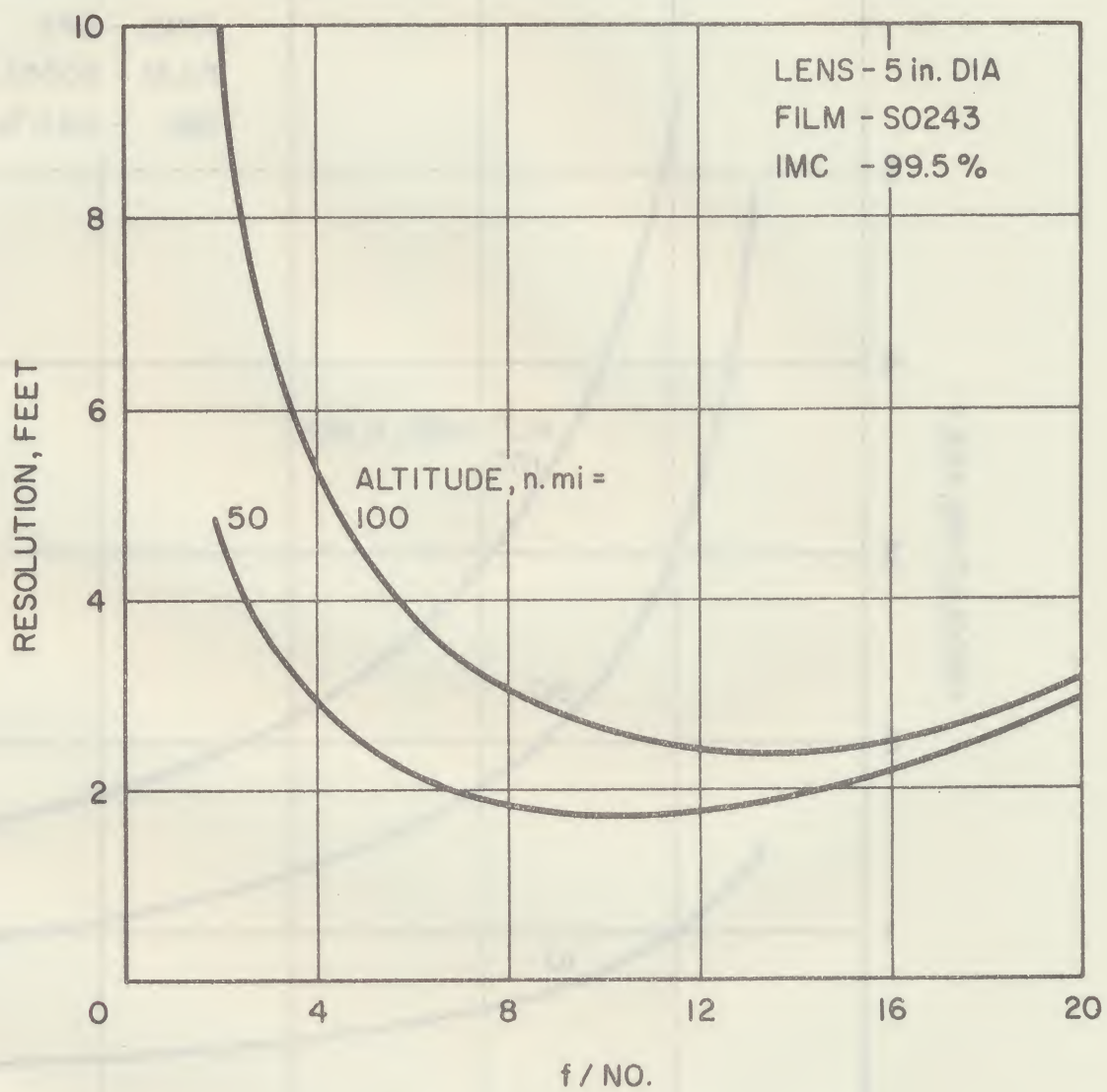


FIGURE 4

GROUND RESOLUTION AS A FUNCTION OF LENS DIAMETER AND ALTITUDE

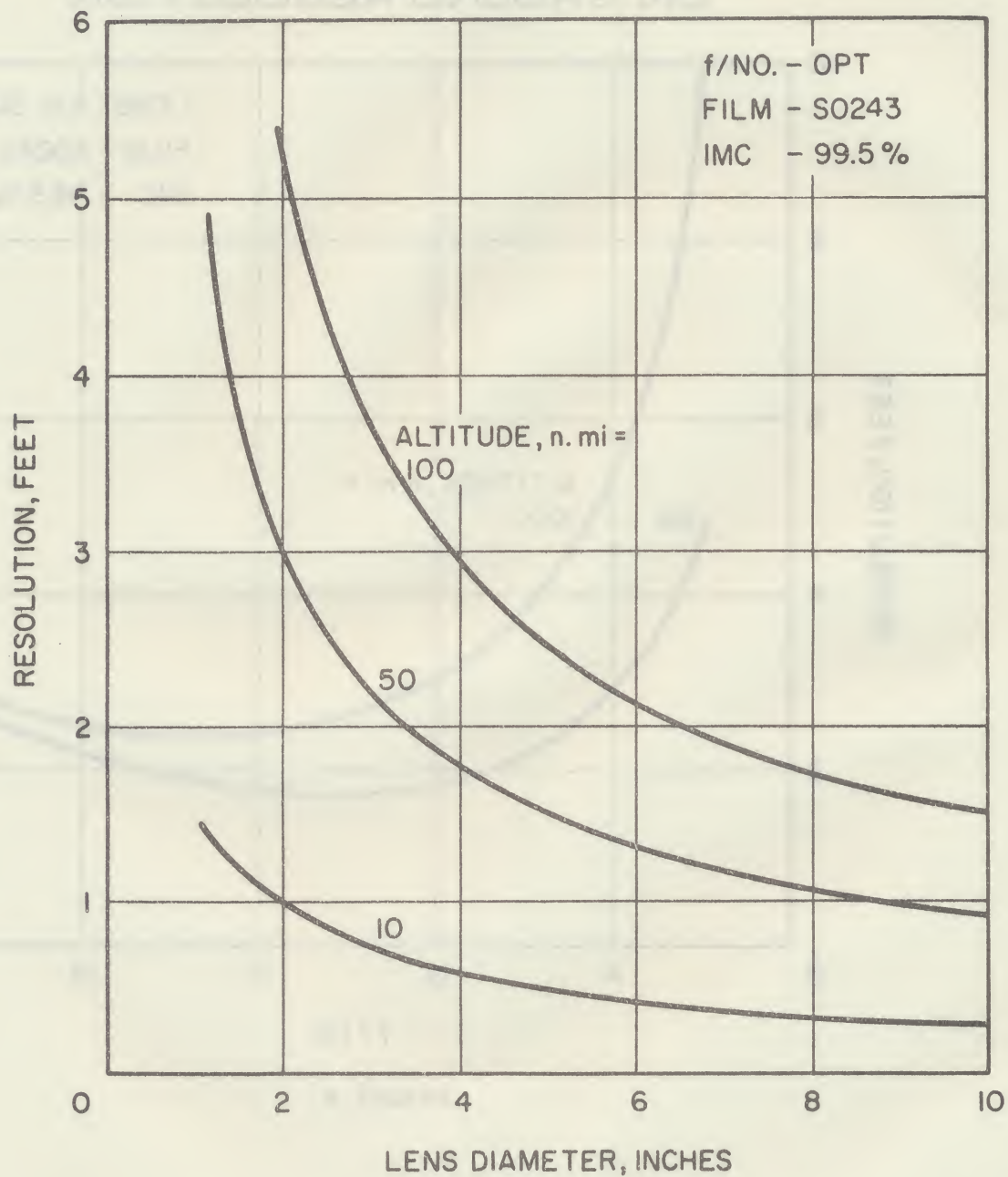


FIGURE 5